S2 S12

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1. A manufacturer produces sweets of length L mm where L has a continuous uniform distribution with range [15, 30].

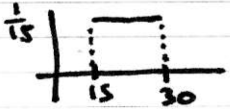
(a) Find the probability that a randomly selected sweet has a length greater than 24 mm. **(2)**

These sweets are randomly packed in bags of 20 sweets.

(b) Find the probability that a randomly selected bag will contain at least 8 sweets with length greater than 24 mm. **(3)**

(c) Find the probability that 2 randomly selected bags will both contain at least 8 sweets with length greater than 24 mm. **(2)**

a) $L \sim U[15, 30)$ $\quad P(x > 24) = \frac{6}{15} = \underline{0.4}$



b) $x \sim B(20, \frac{6}{15})$ $\quad P(x \geq 8) = 1 - P(x \leq 7) = 0.5841$

b) $0.5841^2 = \underline{0.341}$
2

2. A test statistic has a distribution B(25, p).

Given that

$$H_0 : p = 0.5 \qquad H_1 : p \neq 0.5$$

(a) find the critical region for the test statistic such that the probability in each tail is as close as possible to 2.5%. **(3)**

(b) State the probability of incorrectly rejecting H_0 using this critical region. **(2)**

$P(x \leq L) \simeq 0.025$ $\qquad P(x \geq H) \simeq 0.025$

$P(x \leq 7) = 0.0216$ $\qquad P(x > H-1) \simeq 0.025$

$P(x \leq 8) = 0.0539$ $\qquad 1 - P(x \leq H-1) \simeq 0.025$

$P(x \leq H-1) \simeq 0.975$

$\therefore$ $\quad \therefore L = 7$ $\qquad P(x \leq 17) = 0.9784$

CR $\{x \leq 7\} \cup \{x \geq 18\}$ $\qquad \therefore H-1 = 17 \therefore \underline{H=18}$

b) ASL $= 0.0216 + 0.0216 = 0.0432$

4.32% chance of incorrectly rejecting H_0

3. (a) Write down two conditions needed to approximate the binomial distribution by the Poisson distribution.

(2)

A machine which manufactures bolts is known to produce 3% defective bolts. The machine breaks down and a new machine is installed. A random sample of 200 bolts is taken from those produced by the new machine and 12 bolts were defective.

(b) Using a suitable approximation, test at the 5% level of significance whether or not the proportion of defective bolts is higher with the new machine than with the old machine. State your hypotheses clearly.

(7)

3a) large n, small p $\approx np \leq 10$

$x \sim B(200, 0.03)$ $np = 6$ $\approx x \sim Po(6)$

$H_0 : \lambda = 6$ $P(x \geq 12)$ $P(X > 11) = 1 - P(X \leq 11)$
$H_1 : \lambda > 6$

$= 0.0201 < 0.05$

∴ there is enough evidence to reject null hypothesis since result is significant
∴ evidence to suggest the proportion of faulty bolts has increased.

4. The number of houses sold by an estate agent follows a Poisson distribution, with a mean of 2 per week.

(a) Find the probability that in the next 4 weeks the estate agent sells,

(i) exactly 3 houses,

(ii) more than 5 houses.

(5)

The estate agent monitors sales in periods of 4 weeks.

(b) Find the probability that in the next twelve of these 4 week periods there are exactly nine periods in which more than 5 houses are sold.

(3)

The estate agent will receive a bonus if he sells more than 25 houses in the next 10 weeks.

(c) Use a suitable approximation to estimate the probability that the estate agent receives a bonus.

(6)

a) $x \sim Po(8)$ $P(x = 3) = \dfrac{e^{-8} \times 8^3}{3!} = 0.0286$

ii) $P(x > 5) = 1 - P(X \leq 5) = 0.8088$

b) $y \sim B(12, 0.8088)$
$P(y = 9)$ $\binom{12}{9} 0.8088^9 \, 0.1912^3 = 0.2277$

c) $t \sim Po(20)$ $\mu = 20$ $\sigma^2 = 20$ $\approx N(20, 20)$

$P(t > 25)$
$P(t \geq 26)$ cc $P(t > 25.5) \simeq P\left(z > \dfrac{25.5 - 20}{\sqrt{20}}\right)$

$\simeq P(z > 1.23) = 1 - \Phi(1.23) = 0.1093$

5. The queueing time, X minutes, of a customer at a till of a supermarket has probability density function

$$f(x) = \begin{cases} \frac{3}{32}x(k-x) & 0 \leqslant x \leqslant k \\ 0 & \text{otherwise} \end{cases}$$

(a) Show that the value of k is 4 **(4)**

(b) Write down the value of E(X). **(1)**

(c) Calculate Var(X). **(4)**

(d) Find the probability that a randomly chosen customer's queueing time will differ from the mean by at least half a minute. **(3)**

a) $\int f(x)dx = 1 \Rightarrow \frac{3}{32}\int_0^k kx - x^2 dx = \frac{3}{32}\left[\frac{kx^2}{2} - \frac{x^3}{3}\right]_0^k$

$= \frac{3}{32}\left(\frac{1}{6}k^3\right) = 1 \Rightarrow k^3 = 64 \therefore \underline{k = 4}$

b) $E(X) = \int_0^4 x f(x)\, dx = \frac{3}{32}\int_0^4 4x^2 - x^3 dx$

$= \frac{3}{32}\left[\frac{4}{3}x^3 - \frac{1}{4}x^4\right]_0^4 = \frac{3}{32}\left(\frac{64}{3}\right) = 2.$

c) $E(X^2) = \int_0^4 x^2 f(x)dx = \frac{3}{32}\int_0^4 4x^3 - x^4 dx$

$= \frac{3}{32}\left[x^4 - \frac{1}{5}x^5\right]_0^4 = \frac{3}{32}\left(\frac{256}{5}\right) = \frac{24}{5}$

$V(X) = E(X^2) - E(X)^2 = \frac{24}{5} - 4 = \underline{\frac{4}{5}}$

d) $P(1.5 < X < 2.5)$

$= \frac{3}{32}\int_{1.5}^{2.5} 4x - x^2 dx = \frac{3}{32}\left[2x^2 - \frac{x^3}{3}\right]_{1.5}^{2.5} = \underline{0.633}$

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6. A bag contains a large number of balls.

65% are numbered 1

35% are numbered 2

A random sample of 3 balls is taken from the bag.

Find the sampling distribution for the range of the numbers on the 3 selected balls. **(6)**

range = 0   1,1,1 $P = 0.65^3$

2,2,2 $P = 0.35^3$

$\therefore P(\text{range} = 0) = 0.3175$

$\therefore$

range	0	1
P	0.3175	0.6825

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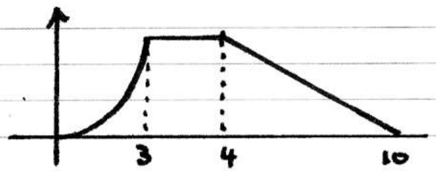
7. The continuous random variable X has probability density function $f(x)$ given by

$$f(x) = \begin{cases} \dfrac{x^2}{45} & 0 \leqslant x \leqslant 3 \\[2ex] \dfrac{1}{5} & 3 < x < 4 \\[2ex] \dfrac{1}{3} - \dfrac{x}{30} & 4 \leqslant x \leqslant 10 \\[2ex] 0 & \text{otherwise} \end{cases}$$

(a) Sketch $f(x)$ for $0 \leqslant x \leqslant 10$ **(4)**

(b) Find the cumulative distribution function $F(x)$ for all values of x. **(8)**

(c) Find $P(X \leqslant 8)$. **(2)**



$0 \leq x \leq 3$ $\quad F(x) = \int_0^x \frac{t^2}{45}\,dt = \left[\frac{t^3}{135}\right]_0^x = \frac{x^3}{135}$

$3 < x < 4$ $\quad F(x) = \int_3^x \frac{1}{5}\,dt + F(3) = \left[\frac{1}{5}t\right]_3^x + \frac{27}{135} = \frac{1}{5}x - \frac{2}{5}$

$4 \leq x \leq 10$ $\quad F(x) = \int_4^x \frac{1}{3} - \frac{t}{30}\,dt + F(4) = \left[\frac{1}{3}t - \frac{t^2}{60}\right]_4^x + \frac{2}{5}$

$= -\frac{x^2}{60} + \frac{1}{3}x - \frac{2}{3}$

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{x^3}{135} & 0 \leq x \leq 3 \\ \frac{1}{5}x - \frac{2}{5} & 3 < x < 4 \\ -\frac{x^2}{60} + \frac{1}{3}x - \frac{2}{3} & 4 \leq x \leq 10 \\ 1 & x > 10 \end{cases}$$

c) $F(8) =$

$\frac{-8^2}{60} + \frac{1}{3}(8) - \frac{2}{3}$

$= \frac{14}{15}$

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8. In a large restaurant an average of 3 out of every 5 customers ask for water with their meal.

A random sample of 10 customers is selected.

(a) Find the probability that

(i) exactly 6 ask for water with their meal,

(ii) less than 9 ask for water with their meal. **(5)**

A second random sample of 50 customers is selected.

(b) Find the smallest value of n such that

$$P(X < n) \geqslant 0.9$$

where the random variable X represents the number of these customers who ask for water. **(3)**

a) x = Customer asks for water $\quad x \sim B(10, 0.6)$

y = Customer does not ask for water

$y \sim B(10, 0.4)$

i) $P(x=6) \Rightarrow P(y=4) = \binom{10}{4} 0.4^4 0.6^6 = 0.251$

ii) $P(x<9) = P(x \leq 8) \Rightarrow P(y \geq 2)$ $\quad P(y > 1)$

$= 1 - P(y \leq 1) = 0.9536$

b) $P(X < n) = P(y > 50 - n) \geq 0.9$ $\quad x \sim B(50, 0.6)$

$\Rightarrow 1 - P(y \leq 50 - n) \geq 0.9$ $\quad y \sim B(50, 0.4)$

$\Rightarrow P(y \leq 50 - n) \leq 0.1$

$P(y \leq 15) = 0.0955 < 0.10$

$P(y \leq 16) = 0.1561 > 0.10$

$\therefore 50 - n = 15 \quad \therefore n = 35$